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Quantized detector networks, particle decays and the quantum Zeno effect

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Abstract
Quantized detector networks (QDN) deal with observers and their apparatus rather than with systems under observation. QDN can be used to investigate the detailed structure of particle decays and the quantum Zeno effect without assuming temporal continuity or invoking non-Hermitian Hamiltonians or complex energies. The formalism is applied to single channel decays, the ammonium molecule and neutral Kaon and B meson decays.

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1. Introduction

There are several interrelated reasons why time is normally assumed to be continuous in standard quantum mechanics (SQM). These cannot be discussed here. On close inspection, however, the continuity of time does not look quite so obvious. The problem is that there are two mutually exclusive views about the nature of observation in physics. These were discussed in detail by Misra and Sudarshan (M&S) in an influential paper on the quantum Zeno effect [1]. On the one hand, no known principle forbids the continuity of time, so the axioms of SQM are stated implicitly in terms of continuous time. When the Schrödinger equation is postulated to be one of them [2], temporal continuity is assumed explicitly. On the other hand, it is an empirical fact that no experiment can actually monitor an SUO (system under observation) in a truly continuous way. The best that could be done in this respect would be to perform a sequence of experiments with a decreasing measurement time scale, in an attempt to see evidence for temporal continuity, such as in the phenomenon known as the quantum Zeno effect [3].

M&S analysed particle decay processes and asked certain questions about them not normally investigated in SQM. Three of these questions were referred to as P, Q and R and this convention will be followed here. $P(0, t; \rho)$ asks for the probability that an unstable system prepared at time zero in state ρ has decayed sometime during the interval $[0, t]$; $Q(0, t; \rho)$ asks for the probability that the prepared state has not decayed during this interval;

and $R(0, t_1, t; \rho)$ asks for the probability that the state has not decayed during the interval $[0, t_1]$, where $0 < t_1 < t$, and has decayed during the interval $[t_1, t]$. M&S stressed that these questions are not what SQM normally calculates, which is the probability distribution of the time at which decay occurs, denoted by q. The difference is that the P, Q and R questions involve a continuous set of observations, or the nearest practical equivalent of it, during each run of the experiment, whereas q involves a set of repeated runs, each with a one-off observation at a different time to determine whether the particle has decayed or not by that time. Because P, Q and R involve a different experimental protocol to q, it should be expected that empirical differences will be observed. Note that the observations M&S refer to can have negative outcomes, i.e., an answer that a particle has *not* decayed by a certain time counts as an observation.

M&S emphasized the limitations of SQM, stressing that although it works excellently in many situations, SQM does not readily give a complete picture of experiments probing questions such as P, Q and R. They concluded that 'there is no standard and detailed theory for the actual coupling between quantum systems and the *classical measuring apparatus*'.

Our approach to quantum mechanics, *quantized detector networks* (QDN) [4–7], attempts to address some of the issues raised by M&S. In QDN, quantum wavefunctions are interpreted as probability amplitudes for signals obtained by observers from physical apparatus, in contrast to SQM, which assumes quantum wavefunctions describe states of SUOs. QDN was motivated primarily by Heisenberg's original vision of quantum mechanics, in which only quantities accessible to an observer are regarded as physically meaningful [8].

QDN is reviewed briefly in the following section, followed by an application to the simplest idealized decay process, a particle decaying via a single channel. The quantum Zeno effect makes an appearance at this point and it is shown that the answer as to whether an SUO decays whilst it is being monitored or whether it remains in its initial state depends on the experimental context, i.e., the details of the apparatus and the measurement protocol involved. Our aim in this paper is to show how complex phenomena such as neutral Kaon decay can be discussed in QDN. To do this we show how to apply QDN to the ammonium molecule and then the techniques developed there are applied to derive the Kaon decay regeneration amplitudes originally discussed by Gell-Mann and Pais in SQM [9]. It will be seen that QDN does not require the introduction of any ad hoc imaginary terms in any energies or the use of non-Hermitian Hamiltonians.

2. The QDN formalism

Time in QDN is correlated with classically certain changes in the observer's information about their apparatus, and because this never happens in a continuous way, as we have emphasized, integers are used rather than reals to represent time. Successive integers do not necessarily represent equal intervals of the observer's physical time.

QDN models apparatus by time-dependent networks of elementary signal detectors (ESDs), each of which gives an unambiguous signal or else a no-signal when examined. The ith ESD in a network at time n is represented by qubit $\mathcal{Q}_n^i$, but should not be automatically identified with any dichotomous physical variable such as electron spin or photon polarization. Individual ESDs do not endure in time, each having an operational function at a single time only. There is no necessary correlation between ESD qubits carrying the same upper index but with different times, i.e., $\mathcal{Q}_n^i$ is unrelated to $\mathcal{Q}_m^i$ when $n \neq m$.

At time n, the r_n ESD qubits in the net at that time constitute a quantum register $\mathcal{R}_n \equiv \mathcal{Q}_n^1 \otimes \mathcal{Q}_n^2 \otimes \cdots \otimes \mathcal{Q}_n^{r_n}$, a tensor product of rank r_n. Such registers contain entangled states as well as separable states. QDN interprets entanglement as a property of *labstates*, the quantum signal states of the apparatus, rather than any property of states of SUOs, *but this does not mean that apparatus itself can be entangled*.

In QDN there is always a natural, preferred basis B_n for $\mathcal{R}_n$, consisting of all possible classical (i.e., sharp) signal states. These are defined in terms of excitations of the *void state*$|0,n)$, the unique state in $\mathcal{R}_n$ for which every detector is in its 'no-signal' state, i.e., $|0,n) \equiv |0,n)_1|0,n)_2\cdots|0,n)_{r_n}$ (henceforth, the tensor product symbol $\otimes$ will be dropped but is implied). The uniqueness of the basis comes from the observer's knowledge about their apparatus.

Associated with each qubit $\mathcal{Q}_n^i$ in $\mathcal{R}_n$ is a *one-signal operator* $\mathbb{A}_{i,n}^+$, which changes the void state $|0,n)_i$ in $\mathcal{Q}_n^i$ to its signal state $|1,n)_i$, leaving all the other qubits unaffected, i.e., $\mathbb{A}_{i,n}^+|0,n) = |0,n)_1\cdots|0,n)_{i-1}|1,n)_i|0,n)_{i+1}\cdots|0,n)_{r_n}$. These operators generate disjoint *signal classes*, consisting of all those elements in B_n created by a given number of distinct signal operators. The zero-signal class consists of the void state $|0,n)$ only, there are r_n one-signal basis states of the form $\mathbb{A}_{i,n}^+|0,n)$ and so on. The r_n+1 distinct signal classes altogether contain all the 2^{r_n} elements of the natural basis B_n, and an arbitrary labstate is a normalized linear combination of any of them. In this paper, only linear combinations of one-signal labstates are needed.

QDN dynamics is described in terms of mappings of labstates from one ESD net to its successor ESD net, which involves mappings between different Hilbert spaces. In order to preserve total probability, QDN uses *Born maps*, which preserve norms. These are insufficient to model all quantum processes, because they are not necessarily linear, so QDN uses *semi-unitary operators*, which are linear Born maps. A semi-unitary operator U from $\mathcal{H}$ into $\mathcal{H}'$ can exist if and only if $\dim\mathcal{H} \leqslant \dim\mathcal{H}'$. Semi-unitarity implies that $U^+U = I$, where I is the identity for $\mathcal{H}$, which means that semi-unitary operators preserve inner products and not just norms.

In the rest of this paper, dynamics will be discussed in terms of finite sequences $\mathbb{U}_{1,0}, \mathbb{U}_{2,1}, \ldots, \mathbb{U}_{n+1,n}, \ldots, \mathbb{U}_{N,N-1}$, $0 < N$, of semi-unitary evolution operators $\mathbb{U}_{n+1,n}$ taking labstates in $\mathcal{R}_n$ to labstates in $\mathcal{R}_{n+1}$ and so on. Such operators satisfy the rule $\mathbb{U}_{n+1,n}^+\mathbb{U}_{n+1,n} = \mathbb{I}_n$, where $\mathbb{I}_n$ is the identity operator in quantum register $\mathcal{R}_n$ [5]. The use of semi-unitary operators implies that $\dim\mathcal{R}_n \leqslant \dim\mathcal{R}_{n+1}$ for $0 \leqslant n < N$. In fact, for particle decay processes, it is necessary to take $\dim\mathcal{R}_n < \dim\mathcal{R}_{n+1}$.

3. One species decays

In this section, QDN is used to describe the quantum physics of what in SQM would be called an unstable particle, the initial state X of which can decay into some multiparticle state Y. At all times total probability will be manifestly conserved. The momenta of the particles will be ignored here, the discussion being designed to illuminate the basic principles of the formalism only.

Typically, the sort of experiment of interest here can be repeated many times, and the formalism gives the quantum description of an ensemble of runs of a basic experiment. Clocks can always be reset, so a typical run of the experiment may be taken to start at time $t = 0$, at which time the observer believes that they have prepared an X state (to use the language of SQM). In QDN, this is represented by the labstate $|\Psi,0) \equiv \mathbb{A}_{X,0}^+|0,0)$, which is automatically normalized to unity.

By time 1, the labstate will have changed from $|\Psi,0)$ to some new labstate $|\Psi,1)$ given by

$$|\Psi,1) = \alpha\mathbb{A}_{X,1}^+|0,1) + \beta\mathbb{A}_{Y_1,1}^+|0,1), \qquad |\alpha|^2 + |\beta|^2 = 1. \tag{1}$$

The outcome possibilities of finding the void state $|0, 1)$ or the two-signal state $\mathbb{A}^+_{X,1}\mathbb{A}^+_{Y_1,1}|0, 1)$ are excluded on dynamical grounds: any run with either of these outcomes would be discounted by the observer as contaminated by external influences (as happens in real experiments). From (1), the amplitude $\mathcal{A}(X, 1|X, 0)$ for the particle not to have decayed by time 1 is given by $\mathcal{A}(X, 1|X, 0) \equiv (0, 1|\mathbb{A}_{X,1}|\Psi, 1) = \alpha$, whilst the amplitude $\mathcal{A}(Y, 1|X, 0)$ for the particle to have made the transition to state Y by time 1 is given by

$$\mathcal{A}(Y, 1|X, 0) \equiv (0, 1|\mathbb{A}_{Y_1,1}|\Psi, 1) = \beta. \tag{2}$$

Total probability is therefore conserved. On the right-hand side of (2), the label Y is itself labelled by a subscript, in this case the number 1, which is the time at which the amplitude is calculated for. The time at which a transition occurs is a crucial feature of the analysis, being directly related to the measurement issues discussed by M&S [1].

The above process conserves signal class, so the dynamics can be discussed economically in terms of the evolution of the signal operators rather than the labstates. For instance, evolution from time 0 to 1 can be given in the form

$$\mathbb{A}^+_{X,0} \to \mathbb{U}_{1,0}\mathbb{A}^+_{X,0}\mathbb{U}^+_{1,0} = \alpha\mathbb{A}^+_{X,1} + \beta\mathbb{A}^+_{Y_1,1}, \tag{3}$$

where $\mathbb{U}_{1,0}$ is a semi-unitary operator satisfying the rule $\mathbb{U}^+_{1,0}\mathbb{U}_{1,0} = \mathbb{I}_0$, with $\mathbb{I}_0$ being the identity for the initial register $\mathcal{R}_0 \equiv \mathcal{Q}^X_0$. The above process involves a change in rank, since $\mathcal{R}_1 \equiv \mathcal{Q}^X_1\mathcal{Q}^{Y_1}_1$. Because $\dim \mathcal{R}_1 > \dim \mathcal{R}_0$, the evolution operator is properly semi-unitary, i.e., satisfies the condition $\mathbb{U}_{1,0}\mathbb{U}^+_{1,0} \neq \mathbb{I}_1$, which is equivalent to irreversibility in SQM.

The description of the next stage of the process, from time 1 to time 2, is more subtle and involves the concept of *null test*, which in SQM is any quantum test which extracts no information from an initial state of an SUO. For example, an electron emerging from a Stern–Gerlach apparatus S_0 in the spin-up state would pass through another Stern–Gerlach apparatus S_1 unscathed, provided the magnetization axis of S_1 was in the same direction as that of S_0.

In QDN, it is not the case that a null test involves no change whatsoever in the observer's information of what is going on. The observer does have the information that time has passed during the null test, and that fact is registered in the observer's memory. Moreover, in QDN, a labstate *always changes in time*, because the quantum register it is in changes with time. What is relevant is the set of components of a labstate relative to the current signal state basis, and it is those components which are related to outcome probabilities. If those components do not change, then the observer may speak about the labstate as being constant in time, but the observer will also have an awareness that the state is evolving in time as well. In other words, the passage of time involves the observer as much as it involves the labstate.

Considering the labstate of the above decay process at time 1, there are now two terms to consider. The first term on the RHS in (3), $\alpha\mathbb{A}^+_{X,1}$, corresponds to a *no decay* outcome by time 1 , and can be regarded as preparing at time 1 an initial X state which could subsequently decay into a Y state or not, with the same dynamical characteristics as for the first stage of the run, held between times 0 and 1. This assumes spatial and temporal homogeneity, a physically reasonable assumption in the absence of gravitational fields and in the presence of suitable apparatus. The second term, $\beta\mathbb{A}^+_{Y_1,1}$, corresponds to *decay having occurred during the first time interval*. Such an outcome is irreversible in this example, but this is not an inevitable assumption in general. Situations where the Y state could revert back to the X state are more complicated but of empirical interest, such as in the ammonium maser and Kaon and B meson decay. These scenarios are discussed later in this paper.

Assuming homogeneity, the next stage of the evolution is given by

$$\mathbb{A}^+_{X,1} \to \mathbb{U}_{2,1}\mathbb{A}^+_{X,1}\mathbb{U}^+_{2,1} = \alpha\mathbb{A}^+_{X,2} + \beta\mathbb{A}^+_{Y_2,2}, \tag{4}$$

$$\mathbb{A}^+_{Y_1,1} \to \mathbb{U}_{2,1}\mathbb{A}^+_{Y_1,1}\mathbb{U}^+_{2,1} = \mathbb{A}^+_{Y_1,2}. \tag{5}$$

Equation (5) is justified as follows. The decay term in (3), proportional to $\mathbb{A}^+_{Y_1,1}$ at time 1, corresponds to the possibility of detecting a decay product state Y at that time. Now there is nothing which requires this information to be extracted precisely at that time. The experimentalist could choose to delay information extraction until some later time, effectively placing the decay product observation 'on hold' . As stated above, this may be represented in SQM by passing a state through a null-test, which does not alter it. In QDN this is represented by equation (5). Essentially, quantum information about a decay is isolated and passed forwards in time until it is physically extracted.

The register $\mathcal{R}_2$ at time 2 has rank 3, being the tensor product $\mathcal{R}_2 = \mathcal{Q}_2^X\mathcal{Q}_2^{Y_1}\mathcal{Q}_2^{Y_2}$. Semi-unitary evolution from time zero to time 2 therefore gives

$$\mathbb{A}^+_{X,0} \to \mathbb{U}_{2,1}\mathbb{U}_{1,0}\mathbb{A}^+_{X,0}\mathbb{U}^+_{1,0}\mathbb{U}^+_{2,1} = \alpha^2\mathbb{A}^+_{X,2} + \alpha\beta\mathbb{A}^+_{Y_2,2} + \beta\mathbb{A}^+_{Y_1,2}, \tag{6}$$

with the various probabilities being read off as the squared moduli of the corresponding terms.

It will be apparent from a close inspection of (6) that what appears to look like a spacetime description with a specific arrow of time is being built up, with a memory of the change of rank of the QDN register at time 1 being propagated forwards in time to time 2. This is represented by the contribution involving $\mathbb{A}^+_{Y_1,2}$, which is interpreted as a potential decay process which may have occurred by time 1, contributing to the overall labstate amplitude at time 2.

Subsequently the process continues in an analogous fashion, with the rank of the register increasing by 1 at each timestep. By time n the dynamics gives

$$\mathbb{A}^+_{X,0} \to \mathbb{U}_{n,0}\mathbb{A}^+_{X,0}\mathbb{U}^+_{n,0} = \alpha^n\mathbb{A}^+_{X,n} + \beta\sum_{k=1}^{n}\alpha^{k-1}\mathbb{A}^+_{Y_k,n}, \tag{7}$$

where $\mathbb{U}_{n,0} \equiv \mathbb{U}_{n,n-1}\mathbb{U}_{n-1,n-2}\cdots\mathbb{U}_{1,0}$ is semi-unitary and satisfies the constraint $\mathbb{U}^+_{n,0}\mathbb{U}_{n,0} = \mathbb{I}_0$. From (7), the survival probability $\Pr(X, n|X, 0)$ that the original state has *not* decayed can be immediately read off and is found to be $\Pr(X, n|X, 0) = |\alpha|^{2n}$. Provided $\beta \neq 0$, this probability appears to fall monotonically with increasing n, which corresponds to particle decay.

The discussion at this point calls for some care with limits, because there arises the theoretical possibility of encountering the quantum Zeno effect, as discussed by M&S [1]. In the following, it will be assumed that $|\alpha| < 1$, because $|\alpha| = 1$ corresponds to a stable particle, which is of no interest here.

Consider the physics of the situation. The calculated probabilities should relate to the observer's physical time t, the clock time used by the observer in the laboratory, which is not assumed here to be a continuous variable on the microscopic level. The temporal label n corresponds to a physical time $t \equiv n\tau$, where τ is some reasonably well-defined time scale characteristic of the apparatus. In the sort of experiments relevant here, τ will be on a minute fraction of a second scale, but certainly nowhere near Planck time scales. The smallest interval that could be achieved in practice would be of the order 10^{-23} s, which is on the shortest hadronic resonance scale, comparable with the time light takes to cross a proton diameter. More realistic measurement scales, involving electromagnetic processes, would be in the 10^{-9} to 10^{-18} s range. Experimentalists would generally have a good understanding of what τ was.

Suppose first that we have some reason to believe that we can relate the transition amplitude α to the characteristic time τ by the rule $|\alpha|^2 \equiv \mathrm{e}^{-\Gamma\tau}$, where Γ is a characteristic inverse time introduced to satisfy this relation. Then the survival probability $P(t_n)$ is given by $P(t) \equiv \Pr(X, n|X, 0) = \mathrm{e}^{-\Gamma t}$, which is the usual exponential decay formula. No imaginary term proportional to Γ in any supposed Hamiltonian or energy has been introduced in order to obtain exponential decay.

A subtlety may arise here however. Exponential decay implies that $|\alpha|^2$ is an analytic function of τ with a Taylor expansion of the form

$$|\alpha|^2 = 1 - \Gamma\tau + O(\tau^2), \tag{8}$$

i.e., one with a nonzero linear term. Under such circumstances, the standard result $\lim_{n\to\infty}(1 - x/n)^n = \mathrm{e}^{-x}$ leads to the exponential decay law. The possibility remains, however, that the dynamics of the apparatus is such that the linear term in (8) is zero, so that the actual expansion is of the form

$$|\alpha|^2 = 1 - \gamma\tau^2 + O(\tau^3), \tag{9}$$

where γ is a positive constant [3]. Then in the limit $n \to \infty$, where $n\tau \equiv t$ is held fixed, the result is given by

$$\lim_{n\to\infty, n\tau=t \text{ fixed}} (1 - \gamma\tau^2 + O(\tau^3))^n = 1, \tag{10}$$

which gives rise to the quantum Zeno effect scenario. An expansion of the amplitude of the form $a = 1 + \mathrm{i}\mu\tau + \nu\tau^2 + O(\tau^3)$ is consistent with (9) for example, if μ is real and $\mu^2 + \nu + \nu^* < 0$.

To understand properly what is going on, it is necessary to appreciate that there are two competing limits being considered: one where an SUO (to use conventional language) is being repeatedly observed over an increasingly large macroscopic laboratory time scale t, and another one where more and more observations are being taken in succession, each separated on a time scale τ which is being brought as close to zero as possible by the experimentalist. In each case, the limit cannot be achieved in the laboratory. The result is that in such experiments, the specific properties of the apparatus and the experimental protocol may play a decisive role in determining the results. If the apparatus is such that (8) holds, then exponential decay will be observed, whereas if the apparatus behaves according to the rule (9), or any reasonable variant of it, then approximations to the quantum Zeno effect should be observed.

The above scenario can be discussed efficiently in terms of semi-unitary matrices. A semi-unitary matrix M is a $r' \times r$ complex matrix such that $\mathsf{M}^+\mathsf{M} = \mathsf{I}_r$, where I_r is the $r \times r$ identity matrix. No semi-unitary matrix exists if $r' < r$.

Consider the X decay scenario discussed above. If the initial labstate is represented by the 1×1 column vector $\Psi_0 \equiv [1]$, then the action of $\mathbb{U}_{1,0}$ acting on the labstate $|\Psi, 0)$ given by (1) may be represented by the action of the 2×1 semi-unitary matrix $\mathsf{U}_{1,0} \equiv [\alpha \quad \beta]^T$ acting on Ψ_0. Then the labstate at time 1 is represented by the 2×1 column vector Ψ_1 given by $\Psi_1 = \mathsf{U}_{1,0}\Psi_0 = [\alpha \quad \beta]^T$. The two required transition amplitudes are just the various components of this vector.

For $n > 0$, the relevant semi-unitary matrix is

$$\mathsf{U}_{n+1,n} = \begin{bmatrix} \alpha & \beta & \mathbf{0}_n^T \\ \mathbf{0}_n & \mathbf{0}_n & I_n \end{bmatrix}^T,$$

where $\mathbf{0}_n$ is a column of n zeros and $\mathbf{0}_n^T$ is its transpose. This leads to the final state

$$\Psi_n = \mathsf{U}_{n,n-1}\mathsf{U}_{n-1,n-2}\cdots\mathsf{U}_{1,0}\Psi_0 = [\alpha^n \quad \beta\alpha^{n-1} \quad \cdots \quad \alpha\beta \quad \beta]^T. \tag{11}$$

The squared modulus of the first component of this column vector gives the same survival probability $|\alpha|^{2n}$ as before. It is also easy to read off all the other transition amplitudes and from them determine discrete time versions of the P, Q and R functions discussed by M&S [1].

Although the QDN analysis gives results which look formally like the standard decay result, the scenario involved is equivalent to that discussed by M&S, namely, there is a constant questioning (or its discrete equivalent) by the apparatus as to whether decay has taken place or not. In this case the results are simple. For Kaon and B meson decays, the results are more complicated.

4. The ammonium system

The explanation by Gell-Mann and Pais [9] of the phenomenon of regeneration in neutral Kaon decay was a successful application of SQM to particle physics. In the standard calculation [10], a non-Hermitian Hamiltonian is used to introduce the two decay parameters needed to describe the observations. We will show that QDN readily reproduces the results of the Gell-Mann and Pais calculation whilst conserving total probability and without the introduction of any complex energies.

The analysis of the Kaon system is more complex than the single particle decay process discussed above, involving the interplay of two distinct neutral Kaons, K^0 and its antiparticle, $\bar{K}^0$. In order to understand the QDN approach to neutral Kaon decay, it will be necessary to review first how systems such as the ammonium molecule are dealt with.

When translation and rotational symmetries are ignored, the ammonium molecule is described in SQM by a superposition of two orthonormal states, each of which represents one of the two possible position states of the single nitrogen atom relative to the plane defined by the three hydrogen atoms. These two states form a basis for a two-dimensional Hilbert space describing the system. Relative to this basis, the Hamiltonian for the system is represented by the Hermitian matrix $\mathsf{H} = \begin{bmatrix} e & f \\ f^* & g \end{bmatrix}$, where e and g are real and f can be complex. If the state of the molecule is represented at time t by the two-component wavefunction $\Psi(t) \equiv [\Psi_1(t)\;\;\Psi_2(t)]^T$, then the Schrödinger equation $\mathrm{i}\hbar d_t \Psi(t) = \mathsf{H}\Psi(t)$ has the general solution $\Psi_j(t) = A_j \mathrm{e}^{-\mathrm{i}\omega^+ t} + B_j \mathrm{e}^{-\mathrm{i}\omega^- t}$, $j = 1, 2$, where A_j and B_j are constants and $\omega^\pm = \frac{1}{2}\{e + g \pm \sqrt{4|f|^2 + (e-g)^2}\}$. This gives time-dependent transition probabilities which are periodic with a frequency given by the difference $\omega^+ - \omega^-$.

In the QDN description, it is assumed that there are two different states, X, Y, with signal operators $\mathbb{A}^+_{X,n}$, $\mathbb{A}^+_{Y,n}$ respectively, evolving according to the rule

$$\begin{aligned} \mathbb{U}_{n+1,n}\mathbb{A}^+_{X,n}|0,n) &= \{a\mathbb{A}^+_{X,n+1} + b\mathbb{A}^+_{Y,n+1}\}|0,n+1), \\ \mathbb{U}_{n+1,n}\mathbb{A}^+_{Y,n}|0,n) &= \{c\mathbb{A}^+_{X,n+1} + d\mathbb{A}^+_{Y,n+1}\}|0,n+1), \end{aligned} \tag{12}$$

where $\mathbb{U}_{n+1,n}$ is a semi-unitary operator. Semi-unitarity requires the constraints $|a|^2 + |b|^2 = |c|^2 + |d|^2 = 1$, $a^*c + b^*d = 0$, which are equivalent to unitarity in SQM in this case. All other states will be disregarded on the basis that there are no dynamical channels between them and states X, Y. With a suitable choice of phases, $\mathbb{U}_{n+1,n}$ can be represented by the semi-unitary matrix $\mathsf{U} = \begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix}$, where an overall possible phase factor is ignored and a and b are as above. The eigenvalues $z^\pm$ of U are given by $z^\pm = \frac{1}{2}\{a + a^* \pm \mathrm{i}\sqrt{4 - (a+a^*)^2}\}$. These are complex conjugates of each other and have magnitude unity, so can be written in the form $z^\pm = \exp\{\pm\mathrm{i}\theta\}$, where θ is real. Writing $a \equiv |a|\,\mathrm{e}^{\mathrm{i}\alpha}$, where α is real, then $\cos\theta = |a|\cos\alpha$. U can always be written in the form $\mathsf{U} = \mathsf{V}\begin{bmatrix} \mathrm{e}^{\mathrm{i}\theta} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta} \end{bmatrix}\mathsf{V}^+$, where $\mathsf{V} = \begin{bmatrix} u & -v^* \\ v & u^* \end{bmatrix}$ and $|u|^2 + |v|^2 = 1$.

It follows that $\mathsf{U}^n = \mathsf{V}\begin{bmatrix} e^{in\theta} & 0 \\ 0 & e^{-in\theta} \end{bmatrix}\mathsf{V}^+$, which gives

$$\begin{aligned} \mathbb{A}^+_{X,0} &\to \{|u|^2\, e^{in\theta} + |v|^2\, e^{-in\theta}\}\mathbb{A}^+_{X,n} + u^* v\{e^{in\theta} - e^{-in\theta}\}\mathbb{A}^+_{Y,n}, \\ \mathbb{A}^+_{Y,0} &\to uv^*\{e^{in\theta} - e^{-in\theta}\}\mathbb{A}^+_{X,n} + \{|u|^2\, e^{-in\theta} + |v|^2\, e^{in\theta}\}\mathbb{A}^+_{Y,n}. \end{aligned} \tag{13}$$

Hence the conditional probabilities are given by

$$\begin{aligned} \Pr(X, n|X, 0) = \Pr(Y, n|Y, 0) &= |u|^4 + |v|^4 + 2|u|^2|v|^2 \cos(2n\theta), \\ \Pr(Y, n|X, 0) = \Pr(X, n|Y, 0) &= 4|u|^2|v|^2 \sin^2(n\theta), \end{aligned} \tag{14}$$

which agrees with the SQM expressions when $2n\theta = (\omega^+ - \omega^-)t$.

It was noted in [3] that a survival probability of the form $P(\tau) \sim 1 - \gamma\tau^2 + O(\tau^3)$ would be needed to make observations of the quantum Zeno effect viable. The above calculation of the ammonium survival probabilities is compatible with this, as can be seen from the expansion $\Pr(X, n|X, 0) = |u|^4 + |v|^4 + 2|u|^2|v|^2 \cos(2n\theta) \sim 1 - 4|u|^2|v|^2 n^2\theta^2 + O\left(n^4\theta^4\right)$. Therefore, it is predicted that the quantum Zeno effect (or at least behaviour analogous to it) should be observable in the ammonium system, if the right experimental conditions are set up. As with the particle decays discussed in the previous section, it would be necessary to ensure that the two limits, $t \to \infty$, $\tau \to 0$, were carefully balanced.

5. Kaon-type decays

More complex systems such as neutral Kaon decay are readily discussed in QDN. Consider three different particle states, X, Y and Z, making transitions between each other in the specific way described below. An important example of such behaviour in particle physics involves the neutral Kaons, with X representing a K^0 meson, Y representing a $\bar{K}^0$ meson and Z representing their various decay products. Kaon decay is remarkable for the phenomenon of regeneration, in which the Kaon survival probabilities fall and then rise with time. More recently, a similar phenomenon has been observed in B meson decay [11].

As before, attention can be restricted to one-signal states. The dynamics is described by the transition rules

$$\mathbb{A}^+_{X,n} \to \alpha\mathbb{A}^+_{X,n+1} + \beta\mathbb{A}^+_{Y,n+1} + \gamma\mathbb{A}^+_{Z_{n+1},n+1}, \tag{15}$$

$$\mathbb{A}^+_{Y,n} \to u\mathbb{A}^+_{X,n+1} + v\mathbb{A}^+_{Y,n+1} + w\mathbb{A}^+_{Z_{n+1},n+1}, \tag{16}$$

$$\mathbb{A}^+_{Z_n,n} \to A^+_{Z_n,n+1}, \tag{17}$$

where semi-unitarity requires the transition coefficients to satisfy the constraints $|\alpha|^2 + |\beta|^2 + \gamma|^2 = |u|^2 + |v|^2 + |w|^2 = 1$, $\alpha^* u + \beta^* v + \gamma^* w = 0$. The above process is a combination of the decay and oscillation processes discussed in previous sections.

The dynamics given by (15)–(17) rules out transitions from Z states to either X or Y states. Therefore, once a Z state is created, it remains a Z state, so there is an irreversible flow from the X and Y states and these eventually disappear. Before that occurs however, there will be back-and-forth transitions between the X and Y states which give rise to the phenomenon of regeneration.

In actual Kaon decay experiments, pure K^0 states can be prepared via the strong interaction process $\pi^- + p \to K^0 + \Lambda$, whilst pure $\bar{K}^0$ states can be prepared via the process $\pi^+ + p \to K^+ + \bar{K}^0 + p$. In our notation, these preparations correspond to initial labstates $\mathbb{A}^+_{X,0}|0, 0)$ and $\mathbb{A}^+_{Y,0}|0, 0)$ respectively. In practice, superpositions of K^0 and $\bar{K}^0$ states may be difficult to prepare directly, but the analysis of Gell-Mann and Pais shows that such states could in principle be prepared indirectly [9]. Therefore, labstates corresponding to X and Y superpositions are physically meaningful and will be used in the following analysis.

Consider an initial labstate of the form $|\Psi, 0) \equiv \left\{x_0\mathbb{A}^+_{X,0} + y_0\mathbb{A}^+_{Y,0}\right\}|0,0)$, where $|x_0|^2 + |y_0|^2 = 1$. Matrix methods are appropriate here. The dynamics of the system will be discussed in terms of the initial column vector $\Psi_0 \equiv [x_0 \quad y_0]^T$, equivalent to the statement that each run of the experiment starts with the rank-2 net register $\mathcal{R}_0 \equiv \mathcal{Q}_0^X\mathcal{Q}_0^Y$. The dynamical rules (15)–(17) map labstates in $\mathcal{R}_0$ into $\mathcal{R}_1 \equiv \mathcal{Q}_1^X\mathcal{Q}_1^Y\mathcal{Q}_1^{Z_1}$, so there is a change of rank from 2 to 3. The transition is represented by the semi-unitary matrix $\mathsf{U}_{1,0} \equiv \begin{bmatrix}\alpha & \beta & \gamma \\ u & v & w\end{bmatrix}^T$, which subsequently generalizes to

$$\mathsf{U}_{n+1,n} \equiv \begin{bmatrix}\alpha & \beta & \gamma & \mathbf{0}_n^T \\ u & v & w & \mathbf{0}_n^T \\ \mathbf{0}_n & \mathbf{0}_n & \mathbf{0}_n & \mathsf{I}_n\end{bmatrix}^T, \qquad n > 0, \tag{18}$$

where I_n is the $n \times n$ identity matrix and $\mathbf{0}_n$ is a column of n zeros. The observer's detector net increases rank by 1 over each time step. The labstate at time t is represented by a column vector Ψ_n with $n + 2$ components, given by $\Psi_n = \mathsf{U}_{n,n-1}\mathsf{U}_{n-1,n-2}\cdots\mathsf{U}_{2,1}\mathsf{U}_{1,0}\Psi_0$. Overall probability is conserved, because of the semi-unitarity of the transition operators.

As before, the key to unravelling the dynamics is linearity, which is guaranteed by the use of semi-unitary evolution operators. Suppose the labstate Ψ_n at time n is represented by $\Psi_n = [x_n \;\; y_n \;\; z_{n,n} \;\; \cdots \;\; z_{1,n}]^T$, where the components x_n and y_n are such that $x_n = \lambda^n x_0$ and $y_n = \lambda^n y_0$, where λ is some complex number to be determined. Such labstates will be referred to as *eigenmodes*. They are not eigenstates of any physical operator, but their first two components, x_n and y_n behave as if they were. The dynamics gives the relations $x_{n+1} = \alpha x_n + u y_n = \lambda x_n$, $y_{n+1} = \beta x_n + v y_n = \lambda y_n$ and $z_{n+1,n+1} = \gamma x_n + w y_n$. Experimentalists will be interested principally in survival probabilities for the X and Y labstates, so the dynamics of Z states will be ignored here, i.e., the behaviour of the components $z_{k,n}$ for $k < n$ will not be discussed. Clearly, however, the QDN formalism is capable of giving much more specific details about the process than just the X and Y survival probabilities.

It will be seen from the above that λ is an eigenvalue of the matrix $\begin{bmatrix}\alpha & u \\ \beta & v\end{bmatrix}$, which means that in principle there are two solutions, λ^+ and λ^-, for the eigenmode values, given by$\lambda^\pm = \frac{1}{2}\{\alpha + v \pm \sqrt{(\alpha - v)^2 + 4\beta u}\}$. It is expected that these will not be mutual complex conjugates in actual experiments, because if they were, the analysis could not explain observed Kaon physics. Therefore, the coefficients α, β, u and v will be such that the above two eigenmode values are complex and of different magnitude and phase, giving rise to two decay channels with different lifetimes, as happens in neutral Kaon decay. In the SQM analysis of neutral Kaon decays, Gell-Mann and Pais described the neutral Kaons as superpositions of two hypothetical particles known as K_1^0 and K_2^0, which are CP eigenstates and have different decay lifetimes [9]. The K_1^0 decays to a two pion state with a lifetime of about 0.9×10^{-10} s whilst the K_2^0 decays to a three pion state with a lifetime of about 0.5×10^{-7} s.

Semi-unitarity guarantees that $|x_{n+1}|^2 + |y_{n+1}|^2 + |z_{n+1,n+1}|^2 = |x_n|^2 + |y_n|^2$, and so it can be deduced that

$$|\lambda|^2 = 1 - \frac{|z_{n+1,n+1}|^2}{|x_n|^2 + |y_n|^2} < 1, \qquad n = 0, 1, 2, \ldots, \tag{19}$$

given $|x_n|^2 + |y_n|^2 > 0$. From this and the conditions $x_n = \lambda^n x_0$, $y_n = \lambda^n y_0$, the eigenmode values can be written in the form $\lambda_1 = \rho_1 \mathrm{e}^{\mathrm{i}\theta_1}$, $\lambda_1 = \rho_2 \mathrm{e}^{\mathrm{i}\theta_2}$, where $\rho_1, \rho_2 < 1$ and θ_1 and θ_2 are real. The eigenmodes at time $t = 0$ corresponding to λ_1 and λ_2 will be denoted by $\Lambda_{1,0}$ and $\Lambda_{2,0}$ respectively, i.e. $\Lambda_{1,0} = [a_1 \;\; b_1]^T$, $\Lambda_{2,0} = [a_2 \;\; b_2]^T$, and then the evolution rules give $\Lambda_{1,n} = [\lambda_1^n a_1 \ \lambda_1^n b_1 \ c_{n,n} \ \cdots \ c_{1,n}]^T$, $\Lambda_{2,n} = [\lambda_2^n a_2 \ \lambda_2^n b_2 \ d_{n,n} \ \cdots \ d_{1,n}]^T$, where the coefficients $\{c_{k,n}\}$, $\{d_{k,n}\}$ can be determined from the dynamics. The initial modes $\Lambda_{1,0}$ and $\Lambda_{2,0}$ are linearly independent provided λ_1 and λ_2 are different. Given that, then any initial labstate Ψ_0 can be expressed uniquely as a normalized linear combination of $\Lambda_{1,0}$ and $\Lambda_{2,0}$, i.e., $\Psi_0 = \mu_1\Lambda_{1,0} + \mu_2\Lambda_{2,0}$, for some coefficients μ_1 and μ_2. This is the analogue of the decompositions $|K^0\rangle = \{|K_1^0\rangle + |K_2^0\rangle\}/\sqrt{2}$, $|\bar{K}^0\rangle = \{|K_1^0\rangle - |K_2^0\rangle\}/\sqrt{2}$ in the Gell-Mann and Pais approach.

From this, the amplitude $\mathcal{A}(X, n|\Psi, 0)$ to find an X signal at time n is given by $\mathcal{A}(X, n|\Psi, 0) = \mu_1 a_1 \lambda_1^n + \mu_2 a_2 \lambda_2^n$, so that the survival probability for X is given by $\Pr(X, n|\Psi, 0) = |\mu_1|^2|a_1|^2\rho_1^{2n} + |\mu_2|^2|a_2|^2\rho_2^{2n} + 2\rho_1^n\rho_2^n \operatorname{Re}\{\mu_1^*\mu_2 a_1^* a_2 \, \mathrm{e}^{-\mathrm{i}(\theta_1-\theta_2)}\}$, and similarly for $\Pr(Y, n|\Psi, 0)$.

There is scope here for various limits to be considered, as discussed in the single channel decay analysis, such that either particle decay is seen or the quantum Zeno effect appears to hold over limited time spans. If we are justified on empirical grounds in writing $\rho_1^n \equiv \mathrm{e}^{-\Gamma_1 t/2}$, $\rho_2^n \equiv \mathrm{e}^{-\Gamma_2 t/2}$, where $t \equiv n\tau$ and Γ_1, Γ_2 correspond to long and short lifetime decay parameters respectively, then the various constants can always be chosen to get full agreement with the standard Kaon survival intensity functions

$$I(K^0) = (\mathrm{e}^{-\Gamma_1 t} + \mathrm{e}^{-\Gamma_2 t} + 2\mathrm{e}^{-(\Gamma_1+\Gamma_2)t/2}\cos\Delta m t)/4, \tag{20}$$

$$I(\bar{K}^0) = (\mathrm{e}^{-\Gamma_1 t} + \mathrm{e}^{-\Gamma_2 t} - 2\mathrm{e}^{-(\Gamma_1+\Gamma_2)t/2}\cos\Delta m t)/4 \tag{21}$$

for pure K^0 decays. Here Δm is proportional to the proposed mass difference between the hypothetical K_1^0 and K_2^0 'particles', which are each CP eigenstates and are supposed to have CP conserving decay channels. From the QDN approach, such objects need not exist. Instead, they are regarded as manifestations of different possible superpositions of K^0 and $\bar{K}^0$ labstates, each of which is physically realizable via the strong interactions, as mentioned above. Conversely, the apparatus dynamics may be such that quantum Zeno-type effects are observed instead of long-term decays. Again, this will depend on the details of the experiment chosen.

6. Concluding remarks

It has been shown here how QDN can give an instrumentalist description of particle decays and the quantum Zeno effect consistent with the spirit of Heisenberg's approach to QM. It provides an alternative description of quantum processes with a novel interpretation of quantum wavefunctions. Avoiding SUOs eliminates some of the problematical concepts associated with QM. Instead of thinking about elementary particles as strange, non-classical objects which sometimes appear to be waves and sometimes particles, we need to think only in terms of how laboratory apparatus responds to physical manipulation by observers. This seems to be a useful and economical way of discussing experimental quantum physics, with a promise of leading to a more dynamical understanding of the relationship between observers and apparatus.

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